B. D. Bojanov, H. A. Hakopian, and A. A. Sahakian, *Spline Functions and Multivariate Interpolations*, Mathematics and Its Applications, Vol. **248**, Kluwer Academic, 1993, ix + 276 pp.

Both spline functions and multivariate interpolation are extensive areas of mathematics and so it cannot be expected that this book is comprehensive. Instead it deals with selected topics in which the authors have conducted research. The first half is on univariate splines, reflecting interests of Bojanov, while the second half comprises interests of the other authors concerning polyhedral splines and multivariate polynomial interpolation. The two halves are fairly distinct, although the first three chapters contain some common introductory material. The emphasis is on theory rather than on practical application, while the exposition is almost self-contained and requires only a basic knowledge of analysis. Each chapter ends with a brief historical discussion and citation of relevant references. Although the authors have made some effort in this respect and have cited some interesting early papers, there are still some surprising omissions.

After three chapters introducing univariate spline functions and the B-spline basis, there is a chapter on total positivity and interpolation. The authors mention both Birkhoff interpolation (interpolation of non-consecutive derivatives) and Birkhoff splines (with continuity imposed on non-consecutive derivatives) but stop short of the full generality of Birkhoff interpolation by Birkhoff splines and do not make clear the elegant duality between the nodes of interpolation and the knots of the splines. The next two chapters deal with natural splines and perfect splines, with their respective optimality properties. The last two chapters on univariate splines concern monosplines, with their connection to quadrature formulae, and periodic splines.

In the second half, the first three chapters are on multivariate B-splines and box splines. Again the material must be selective, but included are all the basic recurrence relations and results on linear independence. The next chapter is on "mean-value interpolation" by multivariate polynomials, which includes Kergin interpolation (a "lifting" of univariate Hermite interpolation) and interpolation of integrals over simplices, which the authors are too modest to refer to as Hakopian interpolation. To my knowledge this is the first time this material has appeared outside scattered research papers, and it therefore seems a pity that it is not dealt with in a more comprehensive manner. The last two chapters continue the study of interpolation by multivariate polynomials and much of the material is the authors' own research. In the first the interpolation is on linear manifolds given as intersections of certain hyperplanes and includes as special cases finite element interpolants on simplices. The final chapter is on multivariate Birkhoff interpolation (see the review of the book "*Multivariate Birkhoff Interpolation*" by R. A. Lorentz, *J. Approx. Theory* **74** (1993), 361).

On the whole this book can be recommended as a clear, readable account of selected topics in approximation theory which will hopefully encourage the reader to delve further into these and related areas.

Tim Goodman

I. Daubechies, *Ten Lectures on Wavelets*, CBMS–NSF Regional Conference Series in Applied Mathematics, Vol. **61**, SIAM, 1992, xix + 357 pp.

Over the past decade, wavelets have become an important tool of mathematical analysis. They have found important applications in several areas of science. On the other hand, it has also become clear that very similar concepts have already been introduced in diverse fields: multigrid methods in numerical analysis, Calderón's formula in operator theory, affine coherent states in quantum physics, subband decompositions in signal processing,... . In that sense, wavelet theory can be viewed as a common language that is well adapted to describing these developments, creating a fruitful interaction between scientists. A good textbook on wavelets faces the difficult task of giving to the reader a clear idea of these pluridisciplinary aspects, while keeping a coherent exposition of the main theoretical results. In that respect, the *Ten Lectures* by I. Daubechies are a brilliant success.

This book describes the different aspects of wavelets, starting from the continuous wavelet transform and its relation with group theory, then concentrating on the redundant sampling of this transform that leads to frames, and finally describing the construction of orthonormal wavelet bases, their applications in functional analysis and operator theory, their related algorithms, and their possible generalization. This natural progression also corresponds to the scientific evolution of the author, who has made important contributions at every step. It gives a strong mathematical unity to this book. Moreover, the author regularly opens "windows" to the different related topics and applications: the auditory model, time–frequency localization, multiresolution approximation, subband coding schemes, subdivision algorithms, ...

The mathematical prerequisite to the reading of this book is a basic knowledge of Fourier analysis and integration theory (some elementary results are recalled in the introduction). As a conclusion, I recommend this book to any scientist who wants to have a clear, yet not simplified, vision of wavelet theory.

ALBERT COHEN

Y. MEYER, *Wavelets. Algorithms & Applications*, translated and revised by R. D. Ryan, SIAM, 1993, xi + 133 pp.

In a short time a large number of papers and books on wavelets have appeared. Each paper and book treats a very particular aspect of the theory or a special application. The novice is therefore faced with the task of going through a rather large amount of technical work. Seldom does he/she find historical pointers, how the theory grew, the variety of applications, and the potential in other areas of scientific interest.

In 1991, Yves Meyer gave a series of lectures on wavelets at the Spanish Institute in Madrid, Spain. The book under review is the result of these lectures (in fact, the English translation of Meyer's lectures). Without doubt the book shows that Meyer has fulfilled the objective of the Spanish Institute: *to present to a scientific audience coming from different disciplines, the prospects that wavelets offer for signal and image processing*, but even more, the book is highly recommended as a very readable introduction to the development from Fourier analysis to the present concept of wavelets.

The first chapter gives a short survey of signal processing and an intuitive view on the development of wavelets. Fourier analysis is the classical tool for stationary signals, but in order to analyze non-stationary or transient signals one needs different techniques including wavelets of *time–frequency* type and wavelets of *time–scale* type. Both types are then briefly explained. In Chapter 2, the author describes seven different origins of wavelet analysis. A problem with (pointwise) convergence of Fourier series led to the search for other orthonormal systems than the trigonometric system, in particular the Haar system and other systems of functions which allow the characterization of certain classes of functions, such as the Hölder spaces C^r. The study of multifractal structures stimulated wavelet techniques since wavelet coefficients can measure the average fluctuations of a signal at different scales, and this holds in particular for Brownian motion. Littlewood and Paley defined dyadic blocks of the Fourier series of a function f to characterize the norm $\|f\|_p$. Zygmund's search in extending the Littlewood–Paley result to n-dimensional Euclidean space led to the introduction of the *mother wavelet* $\psi(x)$. Other developments in the 1930's, such as the Franklin system and the wavelets of Lusin, are also important directions in the new theory of wavelets.